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Semileptonic $B_s \to D_{sJ}(2460) l\nu$ decay in QCD

T.M. Aliev[a,b], K. Azizi[c], A. Ozpineci[d]

Physics Department, Middle East Technical University, Inonu Bulvari, 06531 Ankara, Turkey

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Abstract. Using three point QCD sum rule methods, the form factors relevant to semileptonic $B_s \to D_{sJ}(2460)\ell\nu$ decay are calculated. The q^2 dependences of these form factors are evaluated and compared with the heavy quark effective theory predictions. The dependence of the asymmetry parameter α, characterizing the polarization of the D_{sJ} meson, on q^2 is studied. The branching ratio of this decay is also estimated, and it is shown that it can easily be detected at LHC.

1 Introduction

Recently, very exciting experimental results have been obtained in charmed hadron spectroscopy. The observation of two narrow resonances with charm and strangeness, $D_{sJ}(2317)$ in the $D_s\pi^0$ invariant mass distribution [1–6], and $D_{sJ}(2460)$ in the $D_s^*\pi^0$ and $D_s\gamma$ mass distributions [2–4, 6–8], has raised discussions of the nature of these states and their quark content [9, 10]. Analysis of $D_{s_0}(2317) \to D_s^*\gamma$, $D_{sJ}(2460) \to D_s^*\gamma$ and $D_{S_J}(2460) \to D_{s_0}(2317)\gamma$ indicates that the quark content of these mesons is probably $\bar{c}s$ [11]. In [11] it is also shown that finite quark mass effects for the c quark give non-negligible corrections.

In the heavy quark limit, the lowest lying positive parity states can be grouped into two doublets: a scalar and a pseudo-vector doublet, dubbed $(0^+, 1^+)$ or (D_{s0}^*, D_{s1}') and another pseudo-vector and a tensor doublet, dubbed $(1^+, 2^+)$ or (D_{s1}, D_{s2}^*). In general, if finite mass effects are taken into account, the 1^+ elements of the two doublets will mix. But in the non-strange case, this mixing is known to be small, $\theta = -6°$ [12]. Since at present we have no knowledge of the mixing of these two states in the charm–strange sector, as a first approximation we will assume that the mixing in the charm–strange system is also small, and in this work we will neglect it.

When LHC begins operation, an abundant number of B_s mesons will be produced creating a real possibility for studying the properties of B_s meson and its various decay channels. One of the possible decay channels of B_s meson is its semileptonic $B_s \to D_{sJ}(2460)l\nu$ decay. Analysis of this decay might yield useful information for understanding the structure of the $D_{sJ}(2460)$ meson.

[a] e-mail: taliev@metu.edu.tr
[b] Permanent address: Institute of Physics, Baku, Azerbaijan
[c] e-mail: kazizi@newton.physics.metu.edu.tr
[d] e-mail: ozpineci@metu.edu.tr

It is well known that the semileptonic decays of heavy flavored mesons are very promising tools for the determination of the elements of the CKM matrix, leptonic decay constants as well as the origin of the CP violation. In semileptonic decays the long distance dynamics is parameterized by transition form factors, the calculation of which is a central problem for these decays.

Obviously, for the calculation of the transition form factors, nonperturbative approaches are needed. Among the nonperturbative approaches, the QCD sum rule method [13] received special attention, because this method is based on the fundamental QCD Lagrangian. This method has been successfully applied to a wide variety of problems in hadron physics (for a review, see [14]). The semileptonic decay $D \to \overline{K}^0 l\nu$ is studied using the QCD sum rules with the three point correlation function in [15]. Then, the semileptonic decays $D^+ \to K^{0*}e^+\nu e$ [16], $D \to \pi l\nu$ [17], $D \to \rho l\bar{\nu}e$ [18, 19] and $B \to D(D^*)l\nu e$ [20] are studied in the same framework. In the present work we study the semileptonic decay of the B_s meson to the positive parity $D_{sJ}(2460)$ meson, i.e, $B_s \to D_{sJ}(2460)\ell\nu$, by the QCD sum rule method. Note that, in [21], the decay $B_s \to D_{s_0}(2317)l\nu$ has been studied using the QCD sum rules.

The paper is organized as follows: in Sect. 2 the sum rules for the transition form factors are calculated; Sect. 3 is devoted to a numerical analysis, a discussion and our conclusions.

2 Sum rules for the $B_s \to D_{sJ}(2460)\ell\nu$ transition form factors

The $B_s \to D_{sJ}$ transition proceeds by the $b \to c$ transition at the quark level. The matrix element for the quark level process can be written

$$M_q = \frac{G_F}{\sqrt{2}} V_{cb}\bar{\nu}\gamma_\mu(1-\gamma_5)l\bar{c}\gamma_\mu(1-\gamma_5)b\,. \tag{1}$$

In order to obtain the matrix elements for the $B_s \to D_{sJ}(2460)\ell\nu$ decay, we need to sandwich (1) between the initial and final meson states. Therefore, the amplitude of the $B_s \to D_{sJ}(2460)\ell\nu$ decay can be written

$$M = \frac{G_\mathrm{F}}{\sqrt{2}} V_{cb} \bar{\nu}\gamma_\mu(1-\gamma_5) l \langle D_{sJ}|\bar{c}\gamma_\mu(1-\gamma_5)b|B_s\rangle . \quad (2)$$

The main problem is the calculation of the matrix element $\langle D_{sJ}|\bar{c}\gamma_\mu(1-\gamma_5)b|B_s\rangle$ appearing in (2). Both the vector and axial vector part of $\bar{c}\gamma_\mu(1-\gamma_5)b$ contribute to the matrix element considered above. From Lorentz invariance and parity considerations, this matrix element can be parameterized in terms of the form factors in the following way:

$$\langle D_{sJ}(p',\varepsilon)|\bar{c}\gamma_\mu\gamma_5 b|B_s(p)\rangle = \frac{f_\mathrm{V}(q^2)}{(m_{B_s}+m_{D_{sJ}})}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta, \quad (3)$$

$$\begin{aligned}&\langle D_{sJ}(p',\varepsilon)|\bar{c}\gamma_\mu b|B_s(p)\rangle \\ &= \mathrm{i}\left[f_0(q^2)(m_{B_s}+m_{D_{sJ}})\varepsilon^*_\mu + \frac{f_+(q^2)}{(m_{B_s}+m_{D_{sJ}})}(\varepsilon^* p)P_\mu \right.\\ &\left. + \frac{f_-(q^2)}{(m_{B_s}+m_{D_{sJ}})}(\varepsilon^* p)q_\mu\right], \end{aligned} \quad (4)$$

where $f_\mathrm{V}(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$ are the transition form factors and $P_\mu = (p+p')_\mu$, $q_\mu = (p-p')_\mu$. In all following discussions, as is customary, we will use the following redefinitions:

$$\begin{aligned} f'_\mathrm{V}(q^2) &= \frac{f_\mathrm{V}(q^2)}{(m_{B_s}+m_{D_{sJ}})}, \quad f'_0(q^2) = f_0(q^2)(m_{B_s}+m_{D_{sJ}}), \\ f'_+(q^2) &= \frac{f_+(q^2)}{(m_{B_s}+m_{D_{sJ}})}, \quad f'_-(q^2) = \frac{f_-(q^2)}{(m_{B_s}+m_{D_{sJ}})}. \end{aligned} \quad (5)$$

For the calculation of these form factors, the QCD sum rule method will be employed. We start by considering the following correlator:

$$\begin{aligned}\Pi^{\mathrm{V,A}}_{\mu\nu}(p^2,p'^2,q^2) &= \mathrm{i}^2\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-\mathrm{i}px}\mathrm{e}^{\mathrm{i}p'y} \\ &\times \langle 0|T\left[J_{\nu D_{sJ}}(y)J^{\mathrm{V,A}}_\mu(0)J_{B_s}(x)\right]|0\rangle, \end{aligned} \quad (6)$$

where $J_{\nu D_{sJ}}(y) = \bar{s}\gamma_\nu\gamma_5 c$, $J_{B_s}(x) = \bar{b}\gamma_5 s$, $J^\mathrm{V}_\mu = \bar{c}\gamma_\mu b$ and $J^\mathrm{A}_\mu = \bar{c}\gamma_\mu\gamma_5 b$ are the interpolating currents of the D_{sJ}, B_s, vector and axial vector currents, respectively.

To calculate the phenomenological part of the correlator given in (6), two complete sets of intermediate states with the same quantum number as the currents $J_{D_{sJ}}$ and J_{B_s} respectively are inserted. As a result of this procedure we get the following representation of the above-mentioned correlator:

$$\begin{aligned}&\Pi^{\mathrm{V,A}}_{\mu\nu}(p^2,p'^2,q^2) = \\ &\frac{\langle 0|J^\nu_{D_{sJ}}|D_{sJ}(p',\varepsilon)\rangle\langle D_{sJ}(p',\varepsilon)|J^{\mathrm{V,A}}_\mu|B_s(p)\rangle\langle B_s(p)|J_{B_s}|0\rangle}{(p'^2-m^2_{D_{sJ}})(p^2-m^2_{B_s})} \\ &+\cdots, \end{aligned} \quad (7)$$

where $\cdots$ represent contributions coming from higher states and the continuum. The matrix elements in (7) are defined in the standard way by

$$\begin{aligned}\langle 0|J^\nu_{D_{sJ}}|D_{sJ}(p',\varepsilon)\rangle &= f_{D_{sJ}}m_{D_{sJ}}\varepsilon^\nu, \\ \langle B_s(p)|J_{B_s}|0\rangle &= -\mathrm{i}\frac{f_{B_s}m^2_{B_s}}{m_b+m_s}, \end{aligned} \quad (8)$$

where $f_{D_{sJ}}$ and f_{B_s} are the leptonic decay constants of the D_{sJ} and B_s mesons, respectively. Using (3), (4) and (8) and performing a summation over the polarization of the D_{sJ} meson, (7) can be written

$$\begin{aligned}\Pi^\mathrm{V}_{\mu\nu}(p^2,p'^2,q^2) &= -\frac{f_{B_s}m^2_{B_s}}{(m_b+m_s)}\frac{f_{D_{sJ}}m_{D_{sJ}}}{(p'^2-m^2_{D_{sJ}})(p^2-m^2_{B_s})} \\ &\times\left[f'_0 g_{\mu\nu}+f'_+P_\mu p_\nu + f'_- q_\mu p_\nu\right] \\ &+\text{excited states}, \\ \Pi^\mathrm{A}_{\mu\nu}(p^2,p'^2,q^2) &= -\mathrm{i}\varepsilon_{\mu\nu\alpha\beta}p'^\alpha p^\beta\frac{f_{B_s}m^2_{B_s}}{(m_b+m_s)} \\ &\times\frac{f_{D_{sJ}}m_{D_{sJ}}}{(p'^2-m^2_{D_{sJ}})(p^2-m^2_{B_s})}f'_\mathrm{V} \\ &+\text{excited states}. \end{aligned} \quad (9)$$

In accordance with the QCD sum rule philosophy, $\Pi_{\mu\nu}(p^2,p'^2,q^2)$ can also be calculated from the QCD side with the help of the operator product expansion (OPE) in the deep Euclidean region $p^2 \ll (m_b+m_c)^2$ and $p'^2 \ll (m_c+m_s)^2$. The theoretical part of the correlator is calculated by means of OPE, and up to operators having dimension $d=6$, it is determined by the bare-loop and the power corrections from the operators with $d=3$, $\langle\overline{\psi}\psi\rangle$, $d=4$, $m_s\langle\overline{\psi}\psi\rangle$, $d=5$, $m_0^2\langle\overline{\psi}\psi\rangle$ and $d=6$, $\langle\overline{\psi}\psi\overline{\psi}\psi\rangle$. In calculating the $d=6$ operator, the vacuum saturation approximation is used to set $\langle\overline{\psi}\psi\overline{\psi}\psi\rangle = \langle\overline{\psi}\psi\rangle^2$. In calculating the bare-loop contribution, we first write the double dispersion representation for the coefficients of the corresponding Lorentz structures appearing in the correlation function as

$$\begin{aligned}\Pi'^{\,\mathrm{per}}_i &= -\frac{1}{(2\pi)^2}\int \mathrm{d}s\,\mathrm{d}s'\frac{\rho_i(s,s',q^2)}{(s-p^2)(s'-p'^2)} \\ &+\text{subtraction terms}. \end{aligned} \quad (10)$$

The spectral densities $\rho_i(s,s',q^2)$ can be calculated from the usual Feynman integral with the help of Cutkosky rules, i.e. by replacing the quark propagators with Dirac delta functions: $\frac{1}{p^2-m^2} \to -2\pi\delta(p^2-m^2)$, which implies that all quarks are real. After standard calculations for the corresponding spectral densities we obtain

$$\begin{aligned}\rho_\mathrm{V}(s,s',q^2) &= N_c I_0(s,s',q^2) \\ &\times\left[m_s+(m_s-m_b)B_1+(m_s+m_c)B_2\right], \\ \rho_0(s,s',q^2) &= N_c I_0(s,s',q^2) \\ &\times\left[8(m_b-m_s)A_1 - 4m_b m_c m_s\right. \\ &+4(m_s-m_b+m_c)m_s^2 \\ &-2(m_s+m_c)(\Delta+m_s^2) \\ &\left.-2(m_s-m_b)(\Delta'+m_s^2)+2m_s u\right], \end{aligned}$$

$$
\begin{aligned}
\rho_+(s,s',q^2) &= N_c I_0(s,s',q^2) \\
&\quad \times [4(m_b-m_s)(A_2+A_3)+2(m_b-3m_s)B_1 \\
&\quad -2(m_c+m_s)B_2-2m_s]\,, \\
\rho_-(s,s',q^2) &= N_c I_0(s,s',q^2) \\
&\quad \times [4(m_b-m_s)(A_2-A_3)-2(m_b+m_s)B_1 \\
&\quad +2(m_c+m_s)B_2+2m_s]\,,
\end{aligned} \tag{11}
$$

where

$$
\begin{aligned}
I_0(s,s',q^2) &= \frac{1}{4\lambda^{1/2}(s,s',q^2)}\,, \\
\lambda(s,s',q^2) &= s^2+s'^2+q^4-2sq^2-2s'q^2-2ss'\,, \\
\Delta' &= \left(s'-m_c^2+m_s^2\right), \\
\Delta &= \left(s-m_b^2+m_s^2\right), \\
u &= s+s'-q^2, \\
B_1 &= \frac{1}{\lambda(s,s',q^2)}[2s'\Delta-\Delta' u]\,, \\
B_2 &= \frac{1}{\lambda(s,s',q^2)}[2s\Delta'-\Delta u]\,, \\
A_1 &= \frac{1}{2\lambda(s,s',q^2)}\left[\Delta'^2 s+2\Delta' m_s^2 s+m_s^4 s+\Delta^2 s'\right. \\
&\quad +2\Delta m_s^2 s'+m_s^4 s'-4m_s^2 ss'-\Delta\Delta' u \\
&\quad \left.-\Delta m_s^2 u-\Delta' m_s^2 u-m_s^4 u+m_s^2 u^2\right], \\
A_2 &= \frac{1}{\lambda^2(s,s',q^2)}\left[2\Delta'^2 ss'+4\Delta' m_s^2 ss'+2m_s^4 ss'\right. \\
&\quad +6\Delta^2 s'^2+12\Delta m_s^2 s'^2+6m_s^4 s'^2-8m_s^2 ss'^2 \\
&\quad -6\Delta\Delta' s'u-6\Delta m_s^2 s'u-6\Delta' m_s^2 s'u-6m_s^4 s'u \\
&\quad \left.+\Delta'^2 u^2+2\Delta' m_s^2 u^2+m_s^4 u^2+2m_s^2 s'u^2\right], \\
A_3 &= \frac{1}{\lambda^2(s,s',q^2)}\left[4\Delta\Delta' ss'+4\Delta m_s^2 ss'\right. \\
&\quad +4\Delta' m_s^2 ss'+4m_s^4 ss'-3\Delta'^2 su-6\Delta' m_s^2 su \\
&\quad -3m_s^4 su-3\Delta^2 s'u-6\Delta m_s^2 s'u-3m_s^4 s'u \\
&\quad +4m_s^2 ss'u+2\Delta\Delta' u^2+2\Delta m_s^2 u^2+2\Delta' m_s^2 u^2 \\
&\quad \left.+2m_s^4 u^2-m_s^2 u^3\right].
\end{aligned} \tag{12}
$$

The subscripts V, 0 and $\pm$ correspond to the coefficients of the structures proportional to $\mathrm{i}\varepsilon_{\mu\nu\alpha\beta}p'^\alpha p^\beta$, $g_{\mu\nu}$ and $\frac{1}{2}(p_\mu p_\nu \pm p'_\mu p_\nu)$, respectively. In (11) $N_c=3$ is the number of colors.

The integration region for the perturbative contribution in (10) is determined from the condition that arguments of the three δ functions must vanish simultaneously. The physical region in the s and s' plane is described by the following inequalities:

$$
\begin{aligned}
-1 &\le \frac{2ss'+(s+s'-q^2)\left(m_b^2-s-m_s^2\right)+\left(m_s^2-m_c^2\right)2s}{\lambda^{1/2}\left(m_b^2,s,m_s^2\right)\lambda^{1/2}(s,s',q^2)} \\
&\le +1\,.
\end{aligned} \tag{13}
$$

For the contribution of power corrections, i.e. the contributions of operators with dimensions $d=3$, 4 and 5, we obtain the following results:

$$
\begin{aligned}
& f_{\mathrm{V}}'^{(3)}+f_{\mathrm{V}}'^{(4)}+f_{\mathrm{V}}'^{(5)} \\
&= \frac{1}{rr'}\langle\bar{s}s\rangle-\frac{m_s}{2}\langle\bar{s}s\rangle\left[\frac{-m_c}{rr'^2}+\frac{m_b}{r'r^2}\right] \\
&\quad +\frac{m_s^2}{2}\langle\bar{s}s\rangle\left[\frac{2m_c^2}{r'^3 r}+\frac{m_b^2+m_c^2-q^2}{r'^2 r^2}+\frac{2m_b^2}{r'r^3}\right] \\
&\quad -\frac{m_0^2}{6}\langle\bar{s}s\rangle\left[\frac{3m_c^2}{r'^3 r}+\frac{3m_b^2}{r'r^3}+\frac{2}{r'r^2}\mathrm{i}\right. \\
&\quad \left.+\frac{2m_b^2+2m_c^2+m_b m_c-2q^2}{r'^2 r^2}\right], \\
& f_0'^{(3)}+f_0'^{(4)}+f_0'^{(5)} \\
&= \frac{(m_b-m_c)^2-q^2}{2rr'}\langle\bar{s}s\rangle \\
&\quad +\frac{m_s}{4}\langle\bar{s}s\rangle\left[\frac{-2m_b m_c^2+m_c m_b^2+m_c^3-m_c q^2}{rr'^2}-\frac{m_c+m_b}{rr'}\right. \\
&\quad \left.+\frac{2m_c m_b^2-m_b^3-m_b m_c^2+m_b q^2}{r'r^2}\right] \\
&\quad +\frac{m_s^2}{16}\langle\bar{s}s\rangle\left\{\frac{-16m_b m_c^3+8m_c^2 m_b^2+8m_c^4-8m_c^2 q^2}{r'^3 r}\right. \\
&\quad +\frac{-16m_b^3 m_c+8m_b^4+8m_c^2 m_b^2-8m_b^2 q^2}{r'r^3} \\
&\quad +\frac{4m_c^2-8m_b m_c+4m_b^2-4q^2}{r'^2 r} \\
&\quad +\frac{4m_c^2-8m_b m_c+4m_b^2-4q^2}{r'r^2}-\frac{8}{r'r} \\
&\quad +\frac{1}{r'^2 r^2}\left[-8m_b^3 m_c-8m_b m_c^3+8m_b m_c q^2+4m_b^4\right. \\
&\quad \left.\left.+8m_c^2 m_b^2+4m_c^4-8m_b^2 q^2-8m_c^2 q^2+4q^4\right]\right\} \\
&\quad -\frac{m_0^2}{12}\langle\bar{s}s\rangle\left[\frac{3m_c^2\left(m_c^2+m_b^2-2m_b m_c-q^2\right)}{r'^3 r}\right. \\
&\quad +3m_b^2\frac{m_c^2+m_b^2-2m_b m_c-q^2}{r'r^3} \\
&\quad +\frac{-3m_b m_c\left(m_c^2+m_b^2-q^2\right)+2\left(m_c^2+m_b^2-q^2\right)^2-2m_c^2 m_b^2}{r'^2 r^2} \\
&\quad +\frac{3m_c(m_c-m_b)+2\left(m_b^2-q^2\right)}{rr'^2} \\
&\quad \left.+\frac{3m_b(-3m_c+m_b)+4\left(m_c^2-q^2\right)}{r^2 r'}-\frac{2}{rr'}\right], \\
& f_+'^{(3)}+f_+'^{(4)}+f_+'^{(5)} \\
&= -\frac{1}{2rr'}\langle\bar{s}s\rangle+\frac{m_s}{4}\langle\bar{s}s\rangle\left[\frac{-m_c}{rr'^2}+\frac{m_b}{r'r^2}\right] \\
&\quad +\frac{m_s^2}{32}\langle\bar{s}s\rangle\left[-\frac{16m_c^2}{r'^3 r}-\frac{16m_b^2}{r'r^3}\right. \\
&\quad \left.+\frac{16}{r'r^2}+\frac{-8m_b^2-8m_c^2+8q^2}{r^2 r'^2}\right] \\
&\quad +\frac{m_0^2}{12}\langle\bar{s}s\rangle\left[\frac{3m_c^2}{r'^3 r}+\frac{3m_b^2}{r'r^3}-\frac{2}{r'r^2}\right. \\
&\quad \left.+\frac{2m_b^2+2m_c^2+m_b m_c-2q^2}{r'^2 r^2}\right],
\end{aligned}
$$

$$
\begin{aligned}
& f_-^{\prime(3)} + f_-^{\prime(4)} + f_-^{\prime(5)} \\
& = \frac{1}{2rr'}\langle\bar{s}s\rangle - \frac{m_s}{4}\langle\bar{s}s\rangle\left[\frac{-m_c}{rr'^2} + \frac{m_b}{r'r^2}\right] \\
& + \frac{m_s^2}{32}\langle\bar{s}s\rangle\left[\frac{16m_c^2}{r'^3r} + \frac{16m_b^2}{r'r^3} + \frac{16}{r'r^2} + \frac{8m_b^2+8m_c^2-8q^2}{r^2r'^2}\right] \\
& - \frac{m_0^2}{12}\langle\bar{s}s\rangle\left[\frac{3m_c^2}{r'^3r} + \frac{3m_b^2}{r'r^3}\right. \\
& \left. + \frac{6}{r'r^2} + \frac{2m_b^2+2m_c^2+m_bm_c-2q^2}{r'^2r^2}\right], \qquad (14)
\end{aligned}
$$

where $r = p^2 - m_b^2$, $r' = p'^2 - m_c^2$. We would like to note that the contributions of operators with $d = 6$ are also calculated. Numerically their contributions to the corresponding sum rules turned out to be very small, and therefore we did not present their explicit expressions. Note also that in the present work we neglect the α_s corrections to the bare loop. For consistency, we also neglect α_s corrections in the determination of the leptonic decay constants f_{B_s} and $f_{D_{sJ}}$.

The QCD sum rules for the form factors f'_V, f'_0, f'_+ and f'_- are obtained by equating the phenomenological expression given in (9) and the OPE expression given by (11)–(14) and applying double Borel transformations with respect to the variables p^2 and p'^2 ($p^2 \to M_1^2, p'^2 \to M_2^2$) in order to suppress the contributions of higher states and continuum:

$$
\begin{aligned}
f'_i(q^2) = & -\frac{(m_b+m_s)}{f_{B_s}m_{B_s}^2}\frac{1}{f_{D_{sJ}}m_{D_{sJ}}}\mathrm{e}^{m_{B_s}^2/M_1^2+m_{D_{sJ}}^2/M_2^2} \\
& \times\left[-\frac{1}{(2\pi)^2}\int_{(m_b+m_s)^2}^{s_0}\mathrm{d}s\right. \\
& \times\int_{(m_c+m_s)^2}^{s'_0}\mathrm{d}s'\rho_i(s,s',q^2)\mathrm{e}^{-s/M_1^2-s'/M_2^2} \\
& \left.+\hat{B}\left(f_i^{(3)}+f_i^{(4)}+f_i^{(5)}\right)\right], \qquad (15)
\end{aligned}
$$

where $i = \mathrm{V}$, 0 and $\pm$, and $\hat{B}$ denotes the double Borel transformation operator. In (15), in order to subtract the contributions of the higher states and the continuum, quark–hadron duality assumption is used, i.e., it is assumed that

$$
\rho^{\text{higher states}}(s,s') = \rho^{\text{OPE}}(s,s')\theta(s-s_0)\theta(s-s'_0). \qquad (16)
$$

In our calculations the following rule for double Borel transformations is used:

$$
\begin{aligned}
\hat{B}\frac{1}{r^m}\frac{1}{r'^n} \to & (-1)^{m+n}\frac{1}{\Gamma(m)}\frac{1}{\Gamma(n)}\mathrm{e}^{-s/m_b^2}\mathrm{e}^{-s'/m_c^2} \\
& \times\frac{1}{(M^2)^{m-1}(M'^2)^{n-1}}. \qquad (17)
\end{aligned}
$$

3 Numerical analysis

In this section we present our numerical analysis for the form factors $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$. From the sum rule expressions of these form factors we see that the condensates, leptonic decay constants of B_s and D_{sJ} mesons, continuum thresholds s_0 and s'_0 and Borel parameters M_1^2 and M_2^2 are the main input parameters. In our further numerical analysis we choose the values of the condensates at a fixed renormalization scale of about 1 GeV. The values of the condensates are [22] $\langle\overline{\psi}\psi|_{\mu=1\text{ GeV}}\rangle = -(240 \pm 10\,\text{MeV})^3$, $\langle\bar{s}s\rangle = (0.8\pm0.2)\langle\overline{\psi}\psi\rangle$ and $m_0^2 = 0.8\,\text{GeV}^2$. The quark masses are taken to be $m_c(\mu = m_c) = 1.275 \pm 0.015\,\text{GeV}$, $m_s(1\,\text{GeV}) \simeq 142\,\text{MeV}$ [23] and $m_b = (4.7 \pm 0.1)\,\text{GeV}$ [22]; also, the meson masses are taken to be $m_{D_{sJ}} = 2.46\,\text{GeV}$ and $m_{B_s} = 5.3\,\text{GeV}$. For the values of the leptonic decay constants of the B_s and D_{sJ} mesons we use the results obtained by a two point QCD sum rules analysis: $f_{B_s} = 209\pm38\,\text{MeV}$ [14] and $f_{D_{sJ}} = 225 \pm 25\,\text{MeV}$ [11]. The threshold parameters s_0 and s'_0 are also determined by the two point QCD sum rules: $s_0 = (35 \pm 2)\,\text{GeV}^2$ [13] and $s'_0 = 9\,\text{GeV}^2$ [11]. The Borel parameters M_1^2 and M_2^2 are auxiliary quantities, and therefore the results of the physical quantities should not depend on them. In the QCD sum rule method, OPE is truncated at some finite order, leaving a residual dependence on the Borel parameters. For this reason, the working regions for the Borel parameters should be chosen such that in these regions the form factors are practically independent of them. The working regions for the Borel parameters M_1^2 and M_2^2 can be determined by requiring that, on the one side, the continuum contribution should be small, and on the other side, that the contribution of the operator with the highest dimension should be small. As a result of the above-mentioned requirements, the working regions are determined to be $10\,\text{GeV}^2 < M_1^2 < 20\,\text{GeV}^2$ and $4\,\text{GeV}^2 < M_2^2 < 10\,\text{GeV}^2$.

In order to estimate the width of $B_s \to D_{sJ} l\nu$, it is necessary to know the q^2 dependence of the form factors $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$ in the whole physical region $m_l^2 \le q^2 \le (m_{B_s} - m_{D_{sJ}})^2$. The q^2 dependence of the form factors can be calculated from the QCD sum rules (for details, see [16, 17]). For extracting the q^2 dependence of the form factors from the QCD sum rules we should consider a range for q^2 in which the correlator function can reliably be calculated. For this purpose we have to stay approximately $1\,\text{GeV}^2$ below the perturbative cut, i.e., up to $q^2 = 8\,\text{GeV}^2$. In order to extend our results to the full physical region, we look for a parameterization of the form factors in such a way that, in the region $0 \le q^2 \le 8\,\text{GeV}^2$, this parameterization coincides with the sum rule prediction. The dependences of the form factors $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$ on q^2 are given in Figs. 1–4, respectively. Our numerical calculations show that the best parameterizations of the form factors with respect to q^2 are as follows:

$$
f_i(q^2) = \frac{f_i(0)}{1+\tilde{\alpha}\hat{q}+\tilde{\beta}\hat{q}^2+\tilde{\gamma}\hat{q}^3+\tilde{\lambda}\hat{q}^4}, \qquad (18)
$$

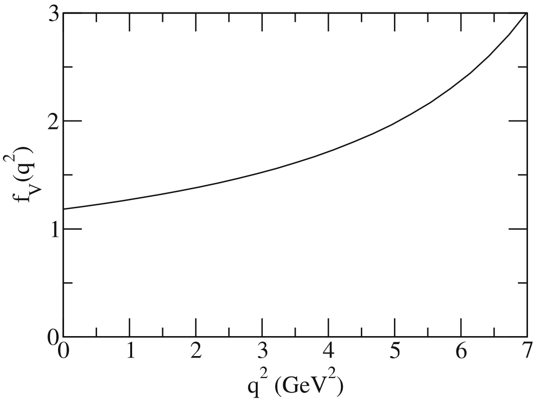

Fig. 1. The dependence of $f_{\rm V}$ on q^2 at $M_1^2 = 15\,{\rm GeV}^2$, $M_2^2 = 6\,{\rm GeV}^2$, $s_0 = 35\,{\rm GeV}^2$ and $s_0' = 9\,{\rm GeV}^2$

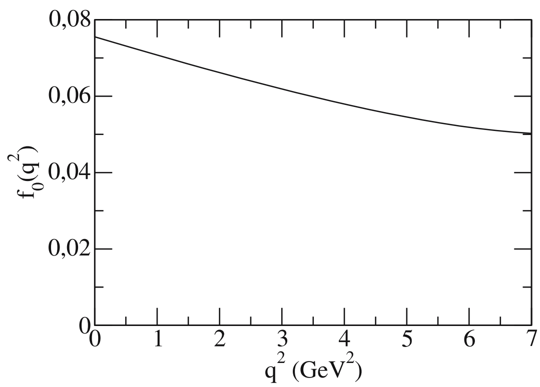

Fig. 2. The dependence of f_0 on q^2 at $M_1^2 = 15\,{\rm GeV}^2$, $M_2^2 = 6\,{\rm GeV}^2$, $s_0 = 35\,{\rm GeV}^2$ and $s_0' = 9\,{\rm GeV}^2$

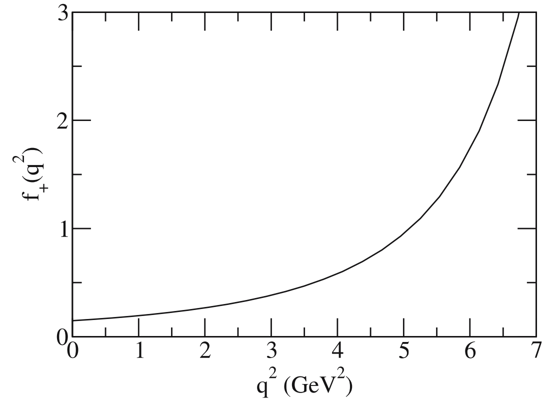

Fig. 3. The dependence of f_+ on q^2 at $M_1^2 = 15\,{\rm GeV}^2$, $M_2^2 = 6\,{\rm GeV}^2$, $s_0 = 35\,{\rm GeV}^2$ and $s_0' = 9\,{\rm GeV}^2$

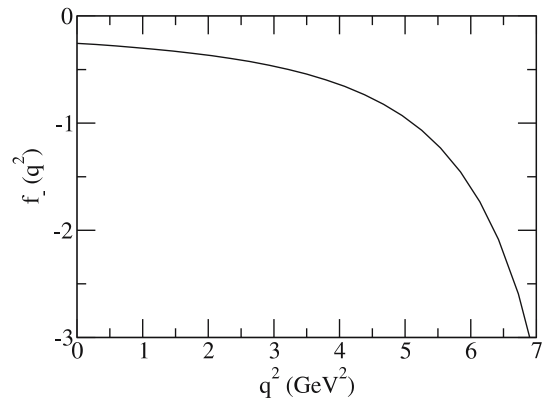

Fig. 4. The dependence of f_- on q^2 at $M_1^2 = 15\,{\rm GeV}^2$, $M_2^2 = 6\,{\rm GeV}^2$, $s_0 = 35\,{\rm GeV}^2$ and $s_0' = 9\,{\rm GeV}^2$

where $\hat{q} = q^2/m_{B_s}^2$. The values of the parameters $f_i(0)$, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\lambda}$ are given in Table 1.

For completeness, let us discuss the heavy quark mass limit of the form factors. In this limit the form factors for the $B_s \to D_{sJ}(2460)$ transition are calculated in [23]. In order to perform the heavy quark mass limit and estimate the dependence of the form factors $f_{\rm V}$, f_0, f_+ and f_- on y, where

$$y = vv' = \frac{m_{B_s}^2 + m_{D_s^*}^2 - q^2}{2m_{B_s}m_{D_s^*}}\,; \qquad (19)$$

we follow the procedure as proposed in [24] and evaluate the sum rules at $q^2 = 0$ by taking $m_b \to \infty$, with $m_c = m_b/\sqrt{z}$, where z is fixed and is given by $\sqrt{z} = y + \sqrt{y^2-1}$ at $q^2 = 0$. Here, v and v' are the four velocities of the B_s and D_{sJ} mesons, respectively. In the $m_b \to \infty$ limit the Borel parameters M_1^2 and M_2^2 take the form $M_1^2 = 2T_1 m_b$ and $M_2^2 = 2T_2 m_b/\sqrt{z}$, where T_1 and T_2 are the new Borel parameters. In this limit, the continuum thresholds s_0 and s_0' become

$$s_0 = m_b^2 + m_b\nu_0\,, \qquad s_0' = \frac{m_b^2}{z} + \nu_0'\frac{m_b}{\sqrt{z}}\,, \qquad (20)$$

Table 1. Parameters appearing in the form factors of the $B_s \to D_{sJ}(2460)\ell\nu$ decay in a four-parameter fit, for $M_1^2 = 15\,{\rm GeV}^2$, $M_2^2 = 6\,{\rm GeV}^2$

	$f(0)$	$\tilde{\alpha}$	$\tilde{\beta}$	$\tilde{\gamma}$	$\tilde{\lambda}$
$f_{\rm V}$	1.18	−1.87	−1.88	−2.41	3.34
f_0	0.076	1.85	0.89	19.0	−79.3
f_+	0.13	−7.14	11.6	21.3	−59.8
f_-	−0.26	−4.11	−3.27	15.2	18.6

and the new integration variables ν and ν' are defined by

$$s = m_b^2 + m_b\nu\,, \qquad s' = \frac{m_b^2}{z} + \nu'\frac{m_b}{\sqrt{z}}\,. \qquad (21)$$

In the $m_b \to \infty$ limit the leptonic decay constants f_{B_s} and $f_{D_{sJ}}$ are rescaled as follows:

$$f_{B_s} = \frac{\hat{f}_{B_s}}{\sqrt{m_b}}\,, \qquad f_{D_{sJ}} = \frac{\hat{f}_{D_{sJ}}}{\sqrt{m_c}}\,. \qquad (22)$$

Taking into account the above-mentioned replacements, the sum rules for the form factors $f_{\rm V}$, f_0, f_+ and f_- in the

$m_b \to \infty$ limit are given by

$$f_{\rm V} = -\frac{z^{1/4}\left(1+\frac{1}{\sqrt{z}}\right)}{\hat{f}_{D_{sJ}}\hat{f}_{B_s}} \mathrm{e}^{(\Lambda+\overline{\Lambda}/T)} \left\{ \frac{-3z^2}{(2\pi)^2(z-1)^3} \times \int_0^{\nu_0} \mathrm{d}\nu \int_0^{\nu_0'} \mathrm{d}\nu'(\nu-\nu')\mathrm{e}^{\frac{-(\nu+\nu')}{2T}}\theta(2y\nu\nu'-\nu^2-\nu'^2) + \langle\overline{\psi}\psi\rangle\left[1+\frac{m_0^2}{T^2}\left(\frac{1}{24}+\frac{1}{12\sqrt{z}}+\frac{\sqrt{z}}{12}\right)\right]\right\}, \quad (23)$$

$$f_0 = -\frac{z^{1/4}}{\hat{f}_{D_{sJ}}\hat{f}_{B_s}\left(1+\frac{1}{\sqrt{z}}\right)} \mathrm{e}^{(\Lambda+\overline{\Lambda}/T)} \left\{ \frac{-3z^{1/2}}{(4\pi)^2(z-1)} \times \int_0^{\nu_0} \mathrm{d}\nu \int_0^{\nu_0'} \mathrm{d}\nu'(\nu-\nu')\mathrm{e}^{\frac{-(\nu+\nu')}{2T}}\theta(2y\nu\nu'-\nu^2-\nu'^2) + \langle\overline{\psi}\psi\rangle\left[\frac{1}{2}+\frac{1}{2z}-\frac{1}{\sqrt{z}} + \frac{m_0^2}{T^2}\left(\frac{-1}{24z^{3/2}}+\frac{1}{12\sqrt{z}}-\frac{\sqrt{z}}{24}\right)\right]\right\}, \quad (24)$$

$$f_+ = -\frac{z^{1/4}\left(1+\frac{1}{\sqrt{z}}\right)}{\hat{f}_{D_{sJ}}\hat{f}_{B_s}} \mathrm{e}^{(\Lambda+\overline{\Lambda}/T)} \left\{ \frac{3z^{3/2}(\sqrt{z}-1)^4(\sqrt{z}+1)^2}{(4\pi)^2(z-1)^5} \times \int_0^{\nu_0} \mathrm{d}\nu \int_0^{\nu_0'} \mathrm{d}\nu'(\nu-\nu')\mathrm{e}^{\frac{-(\nu+\nu')}{2T}}\theta(2y\nu\nu'-\nu^2-\nu'^2) + \langle\overline{\psi}\psi\rangle\frac{m_0^2}{T^2}\left[\frac{1}{12}+\frac{1}{24\sqrt{z}}+\frac{\sqrt{z}}{24}\right]\right\}, \quad (25)$$

$$f_- = -\frac{z^{1/4}\left(1+\frac{1}{\sqrt{z}}\right)}{\hat{f}_{D_{sJ}}\hat{f}_{B_s}} \mathrm{e}^{(\Lambda+\overline{\Lambda}/T)} \left\{ \frac{3z^{3/2}(\sqrt{z}+1)^4(\sqrt{z}-1)^2}{(4\pi)^2(z-1)^5} \times \int_0^{\nu_0} \mathrm{d}\nu \int_0^{\nu_0'} \mathrm{d}\nu'(\nu-\nu')\mathrm{e}^{\frac{-(\nu+\nu')}{2T}}\theta(2y\nu\nu'-\nu^2-\nu'^2) - \langle\overline{\psi}\psi\rangle\frac{m_0^2}{T^2}\left[\frac{1}{12}+\frac{1}{24\sqrt{z}}+\frac{\sqrt{z}}{24}\right]\right\}, \quad (26)$$

In deriving these results we take $T_1 = T_2 = T$, and the parameters Λ and $\overline{\Lambda}$ are obtained from the two point sum rules, leading to the prediction $\Lambda = 0.62$ GeV [25] and $\overline{\Lambda} = 0.86$ GeV [26]. Numerical analysis of the above sum rules gives

$$f_{\rm V}(y_{\max}) = 0.4\,, \qquad f_0(y_{\max}) = 0.031\,, \qquad f_+(y_{\max}) = 0.15\,, \qquad f_-(y_{\max}) = -0.27\,, \quad (27)$$

where $y_{\max} = 1.30931$, which corresponds to $q^2 = 0$. When we compare these results with the ones given in (18) at $q^2 = 0$, we see that finite mass corrections are essential for $f_{\rm V}(q^2)$ and $f_0(q^2)$. Here we note that the HQET limit for the transition form factors for B decays is discussed in [27].

For $B_s \to D_{sJ}(2460) l\nu$ decay it is also possible to determine the polarization of the $D_{sJ}(2460)$ meson. To this aim we determine the asymmetry parameter α, characterizing the polarization of the $D_{sJ}(2460)$ meson, as follows:

$$\alpha = 2\frac{\mathrm{d}\Gamma_{\rm L}/\mathrm{d}q^2}{\mathrm{d}\Gamma_{\rm T}/\mathrm{d}q^2} - 1\,, \quad (28)$$

where $\mathrm{d}\Gamma_{\rm L}/\mathrm{d}q^2$ and $\mathrm{d}\Gamma_{\rm T}/\mathrm{d}q^2$ are the differential widths of the decay to the states with longitudinally and transversally polarized $D_{sJ}(2460)$ meson. After some calculations for the differential decay rates $\mathrm{d}\Gamma_{\rm L}/\mathrm{d}q^2$ and $\mathrm{d}\Gamma_{\rm T}/\mathrm{d}q^2$ we get

$$\frac{\mathrm{d}\Gamma_{\rm T}}{\mathrm{d}q^2} = \frac{1}{8\pi^4 m_{B_s}^2}|\mathbf{p}'|G_{\rm F}^2|V_{cb}|^2 \times\left\{(2A+Bq^2)\left[|f_{\rm V}'|^2\left(4m_{B_s}^2|\mathbf{p}'|^2\right)+|f_0'|^2\right]\right\}, \quad (29)$$

$$\begin{aligned}\frac{\mathrm{d}\Gamma_{\rm L}}{\mathrm{d}q^2} = {}& \frac{1}{16\pi^4 m_{B_s}^2}|\mathbf{p}'|G_{\rm F}^2|V_{cb}|^2 \\ & \times\Big\{\left(2A+Bq^2\right)\Big[|f_{\rm V}'|^2\big(4m_{B_s}^2|\mathbf{p}'|^2 \\ & + m_{B_s}^2\frac{|\mathbf{p}'|^2}{m_{D_{sJ}}}\left(m_{B_s}^2-m_{D_{sJ}}^2-q^2\right)\big)+|f_0'|^2 \\ & -|f_+'|^2\frac{m_{B_s}^2|\mathbf{p}'|^2}{m_{D_{sJ}}^2}\left(2m_{B_s}^2+2m_{D_{sJ}}^2-q^2\right) \\ & -|f_-'|^2\frac{m_{B_s}^2|\mathbf{p}'|^2}{m_{D_{sJ}}^2}q^2 - 2\frac{m_{B_s}^2|\mathbf{p}'|^2}{m_{D_{sJ}}^2} \\ & \times\left(\mathrm{Re}\left(f_0'f_+'^* + f_0'f_-'^* + \left(m_{B_s}^2-m_{D_{sJ}}^2\right)f_+'f_-'^*\right)\right)\Big] \\ & -2B\frac{m_{B_s}^2|\mathbf{p}'|^2}{m_{D_{sJ}}^2}\Big[|f_0'|^2+\left(m_{B_s}^2-m_{D_{sJ}}^2\right)^2|f_+'|^2 \\ & +q^4|f_-'|^2+2\left(m_{B_s}^2-m_{D_{sJ}}^2\right)\mathrm{Re}\left(f_0'f_+'^*\right)+2q^2f_0'f_-'^* \\ & +2q^2\left(m_{B_s}^2-m_{D_{sJ}}^2\right)\mathrm{Re}\left(f_+'f_-'^*\right)\Big]\Big\}\,, \end{aligned} \quad (30)$$

where

$$\begin{aligned} |\mathbf{p}'| &= \frac{\lambda^{1/2}\left(m_{B_s}^2, m_{D_{sJ}}^2, q^2\right)}{2m_{B_s}}\,, \\ A &= \frac{1}{12q^2}\left(q^2-m_l^2\right)^2 I_0\,, \\ B &= \frac{1}{6q^4}\left(q^2-m_l^2\right)\left(q^2+2m_l^2\right)I_0\,, \\ I_0 &= \frac{\pi}{2}\left(1-\frac{m_l^2}{q^2}\right). \end{aligned} \quad (31)$$

The dependence of the asymmetry parameter α on q^2 is shown in Fig. 5. From this figure we see that the asymmetry parameter α varies between -0.3 and 0.3 when q^2 lies in the region $m_l^2 \le q^2 \le 6\,\mathrm{GeV}^2$. An interesting observation is that around $q^2 = 5.2\,\mathrm{GeV}^2$ the asymmetry parameter changes sign. Therefore, measurement of the polarization asymmetry parameter α at fixed values of q^2 and determination of its sign can give unambiguous information on the quark structure of the D_{sJ} meson.

Having come to the end of this section we would like to present the value of the branching ratio of this decay. Taking into account the q^2 dependence of the form factors and performing the integration over q^2 in the limit $m_l^2 \le q^2 \le (m_{B_s} - m_{D_{sJ}})^2$, and using the total life-time $\tau_{B_s} = 1.46\times10^{-12}$ s [28], we get for the branching ratio

$$\mathcal{B}(B_s \to D_{sJ}(2460)\ell\nu) \simeq 4.9\times10^{-3}\,, \quad (32)$$

which might easily be measurable at LHC.

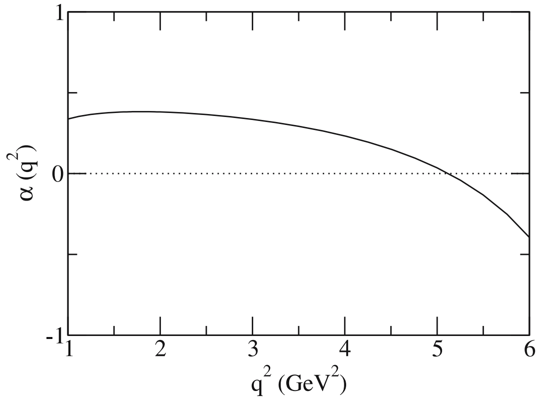

Fig. 5. The dependence of α on q^2

In conclusion, the semileptonic $B_s \to D_{sJ}(2460)\ell\nu$ decay is investigated by the QCD sum rule method. The q^2 dependences of the transition form factors are evaluated. The dependence of the asymmetry parameter α on q^2 is investigated, and the branching ratio is estimated to be measurably large at LHC.

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